



# TARGET TRACKING IN THREE-DIMENSIONAL COORDINATES BY COMBINING RADAR AND IRST DATA

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## ABSTRACT

Since the tracking of moving targets is a vital issue in military and civilian applications, aerospace industries are always looking for accurate, low-error, computationally light, and uncomplicated algorithms for target tracking. Today, most modern military systems are equipped with various sensors. The ideal operation of such sensors helps to realize target tracking. Due to the nature of the sensor system and the types of noises, one kind of sensor alone cannot be ideally used in target tracking. As a result, several different sensors are used in new systems for tracking. Radar systems are usually used to measure the angle and range of targets. Although they measure the range with high accuracy, radar systems cannot measure the target angle with proper accuracy. On the other hand, IRST data can measure the target angle with high accuracy and determine the direction of the target completely, but they do not provide special information about the target range. Providing a structure to integrate the information of these sensors facilitates the exact location of the target and also tracks the change of the target location. In this study, various structures for target tracking in 3D coordinates have been presented by combining radar data and IRST data. The results of target tracking with radar or IRST are very weak compared to the combination of radar and IRST. The performance of the SVF algorithm is favorable in terms of calculation speed and implementation complexity, as expected. Also, IRST-based tracking alone is expected to require less time and subsequently show poorer performance.

**Keywords:** Kalman filter, tracking algorithm, interacting multiple models (IMM), IRST

## INTRODUCTION

Computer-aided multiple target tracking (MTT) is very common in radar surveillance systems and has been proposed as a new branch of research in many studies [1-5]. The basic principles

of MTT were proposed in 1655 by Wax [6]. The next breakthrough in MTT theory was realized in 1964 by Sittler [7] and set the stage for further developments. After the introduction of the Kalman filter in the early 1970s, Bar-Shalom [8-9] and Singer [10-11] established a new direction in MTT theory by combining data allocation and the Kalman filter.

The first and most famous application of MTT was the track-while scan (TWS) system described in Hovanssian's book [12]. The TWS system is a special sub-branch of the MTT system in which data is received in the form of a regular time sequence through a regular sensor sweep. For common TWS systems, the search and update operations are performed simultaneously. At a constant rate, a sensor monitors new targets and tracks targets with the same observation time, the same detection threshold, and the same waveform. TWS systems only retain traces within the system's pre-defined search range.

Due to the needs of the modern world and solving the challenge of detecting and extracting the desired parameters from the target, the phased array radar technology has been considered since 1960. From 1980 onwards, phased array radar was widely used for military and civilian applications (tracking satellites in the field of aerospace surveillance, meteorology, etc.). Alkiori et al. (1991) used the Kalman filter to estimate the bias of tracking systems [13]. Yako et al. (1993) modeled tracking as an estimation problem and formulated its relations [14]. Saha (1996) used composite structures in tracking systems [15]. Blackman (1999) introduced the basic principles of designing tracking systems in detail. The resulting article is one of the most important and fundamental studies in this field [16]. Vosferg (2008) generalized the Alkiori scheme for a telecommunication system to optimize the Kalman filter [17]. Chengu et al. (2003) used the combination of radar information with synthesis aperture radar (SAR) images to track targets [18]. Naida (2009) combined radar information with an optical sensor and used it to track moving targets. The results of the study were published in the form of a comprehensive article in the field of goal tracking [19]. Jian Xou (2012) used the combination of information to track targets in wireless systems and proved the possibility of using the combination of information in wireless systems [20].

From 1999 onwards, different algorithms have been presented for displaying the moving target (MTI) in fuzzy arrays and predicting the trajectory of the target. The process of evolution and optimization of these algorithms continues. Interacting multiple model (IMM) algorithms were very much considered in this field due to their effectiveness, good performance, and reduction of calculations. From 2008 onwards, various algorithms were extracted and opened a new field for fast and low-error algorithms. In the multiple model approach [21], several models are assumed for the movement of targets. A filter is considered for each model based on the likelihood function. The probability of the correctness of each model is calculated to define the dynamic target.

H.A.P.Blom and Y.Bar-Shalom (1988) [22] invented interacting multiple models (IMM) as the most effective method proposed for the estimation of hybrid systems. A hybrid system is appropriately described by continuous values of the state space and a set of model states. Changing the mode or switching between different models is done randomly. In [22], a complementary procedure for the previous multi-model approach is proposed. This IMM algorithm interacts with multiple models using a set of tracking filters. Instead of the filters working independently, the models interact through probabilities. Due to interaction, individual filters adjust their parameters and provide optimal output based on their input. To determine the final output of the system, the weighted average of the output of individual filters is calculated. The weighting factors are part of the filter formulas. Also, this approach does not need to use a separate maneuver detector like the first method. In [23], extensive studies have been done on IMM methods for target tracking. Due to the prevalence of multiple model applications in radar systems, the details of this method are explained below.

Considering the novelty of phased array radar and optimal tracking algorithms in new technologies and Iran's high-tech industries, in this thesis, different tracking methods are

investigated in this field. The key purpose is to find the best algorithm with the least computational complexity and higher accuracy in tracking targets. To achieve more accuracy in tracking, small- and large-time intervals are considered for maneuvering movements and non-maneuvering movements or constant speed, respectively. The performance of the  $\gamma$ - $\beta$ - $\alpha$  filter, Kalman filter (KF), and IMM algorithms were compared. It was also shown that in tracking moving targets with heavy maneuvers such as satellites, the IMM algorithm has better performance than the KF and  $\gamma$ - $\beta$ - $\alpha$  filters. Also, the Kalman filter and the extended Kalman filter have better performance than the  $\gamma$ - $\beta$ - $\alpha$  filter.

**A proposed method for combining radar data and IRST data**

In this section, six different schemes for combining radar data and IRST data for target tracking in 3D Cartesian coordinates are presented. IRST and radar measurements were made in polar coordinates and refer to IRST and radar measurements, respectively. IRST noise covariance matrix is calculated as follows.

$$R_i = \begin{bmatrix} \sigma_{i\theta}^2 & 0 \\ 0 & \sigma_{i\varphi}^2 \end{bmatrix} \tag{1}$$

The radar noise covariance matrix is calculated as Relation (2).

$$R_r = \begin{bmatrix} \sigma_{r\theta}^2 & 0 & 0 \\ 0 & \sigma_{r\varphi}^2 & 0 \\ 0 & 0 & \sigma_{rr}^2 \end{bmatrix} \tag{2}$$

In the measurement variable selection (SM) algorithm, the measurement vectors are formed through the appropriate selection of radar and IRST data. The measurement vector selects angular measurement and range measurement from IRST data and radar data, respectively. Similarly, the covariance matrix is calculated. The general structure of the proposed method is represented in Figure (1).

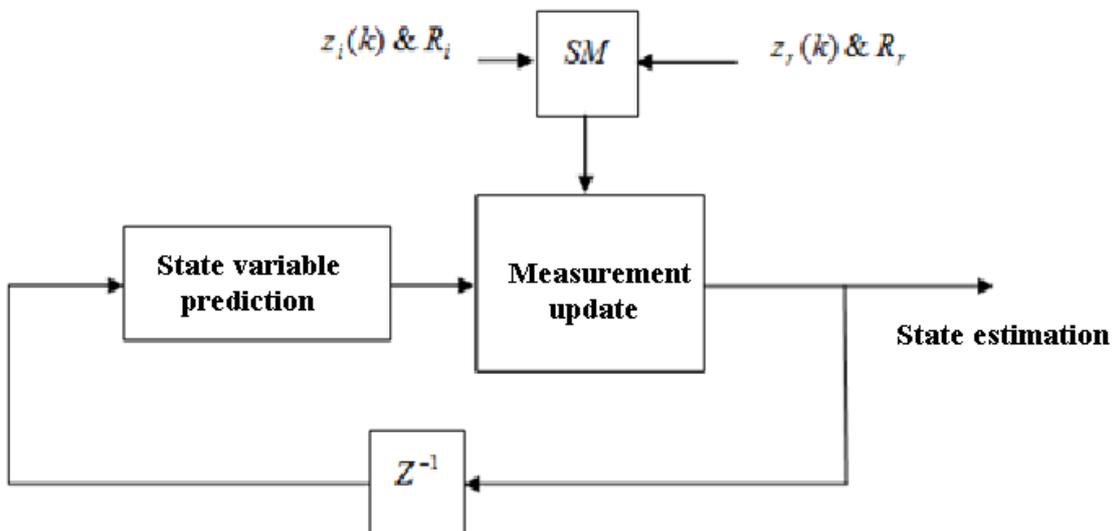


Figure 1: Block diagram of SM algorithm

This structure is a simple extended Kalman filter, and the equations required to apply the tracking algorithm are compiled as follows.

The state prediction equations are compiled as follows.

$$\begin{aligned}\tilde{X}(k|k-1) &= F\hat{X}(k-1|k-1) \\ \tilde{P}(k|k-1) &= F\hat{P}(k-1|k-1)F^T + GQG^T\end{aligned}\quad (3)$$

The combination of the resulting variables is as follows.

$$z(k) = [\theta_i \ \varphi_i \ r_r]^T \quad (4)$$

The variance matrix is formed as follows.

$$R = \begin{bmatrix} \sigma_{i\theta}^2 & 0 & 0 \\ 0 & \sigma_{i\varphi}^2 & 0 \\ 0 & 0 & \sigma_{rr}^2 \end{bmatrix} \quad (5)$$

The measurement update matrix is calculated as follows.

$$\begin{aligned}H &= h(\tilde{X}(k|k-1)) \\ \tilde{z}(k|k-1) &= H\tilde{X}(k|k-1) \\ e &= z(k) - \tilde{z}(k|k-1) \\ S &= H\tilde{P}(k|k-1)H^T + R \\ K &= \tilde{P}(k|k-1)H^T + S^{-1} \\ \hat{X}(k|k) &= \tilde{X}(k|k-1) + ke \\ \hat{P}(k|k) &= [I - KH]\tilde{P}(k|k-1)\end{aligned}\quad (6)$$

In the combination of measurement variables (MF) algorithm, the measurement vectors are the result of a suitable combination of radar data and IRST data. The measurement vector extracts the angular measurement (horizontal and vertical) from the combination of IRST and radar data. It also selects range measurements from radar data only. Similarly, the measurement covariance matrix is calculated. The general structure of the process is shown in Figure (2). Instead of combining the measurement data, in the developed Kalman filter, the IRST and radar measurements were combined in the form of an added measurement vector, and the measurement noise variances are related to both sensors to produce similar results.

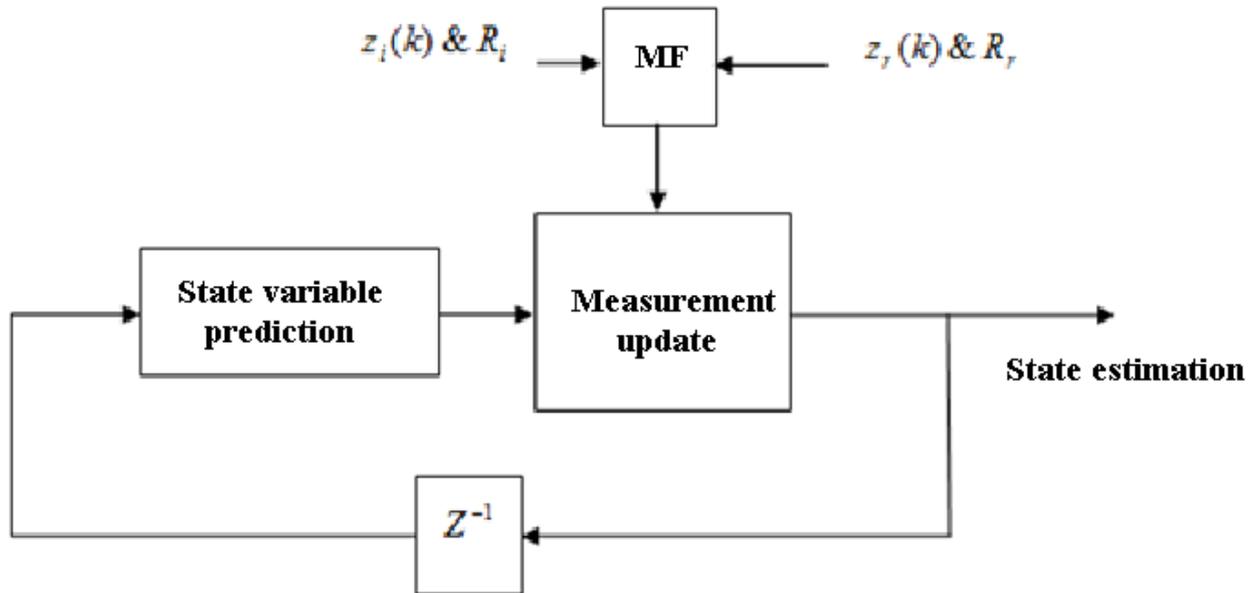


Figure 2: Block diagram of MF algorithm

The state prediction and measurement update equations in this structure are similar to the previous structure (Relations 5 and 6). The combined Relations in this structure are compiled as follows.

$$z(k) = [\theta_r \ \varphi_r]^T \tag{7}$$

$$R_d = \begin{bmatrix} \sigma_{r\theta}^2 & 0 \\ 0 & \sigma_{r\varphi}^2 \end{bmatrix}$$

$$z_f = z_i(k) + z_i(k)[R_i + R_d]^{-1}(z_d(k) - z_i(k))$$

$$R_f = R_i - R_i[R_i + R_d]^{-1}R_i$$

$$z(k) = [z_f(1) \ z_f \ r_r]^T$$

$$R = \begin{bmatrix} R_f(1,1) & 0 & 0 \\ 0 & R_f(1,1) & 0 \\ 0 & 0 & \sigma_{rr}^2 \end{bmatrix}$$

In the state vector combination (SVF) algorithm shown in Figure (3), the trajectories are determined separately by radar and IRST measurements. For target final state calculations, the resulting state path vectors are combined. Similarly, the linear state covariances of the individual paths are combined to form the final state error covariance matrix.

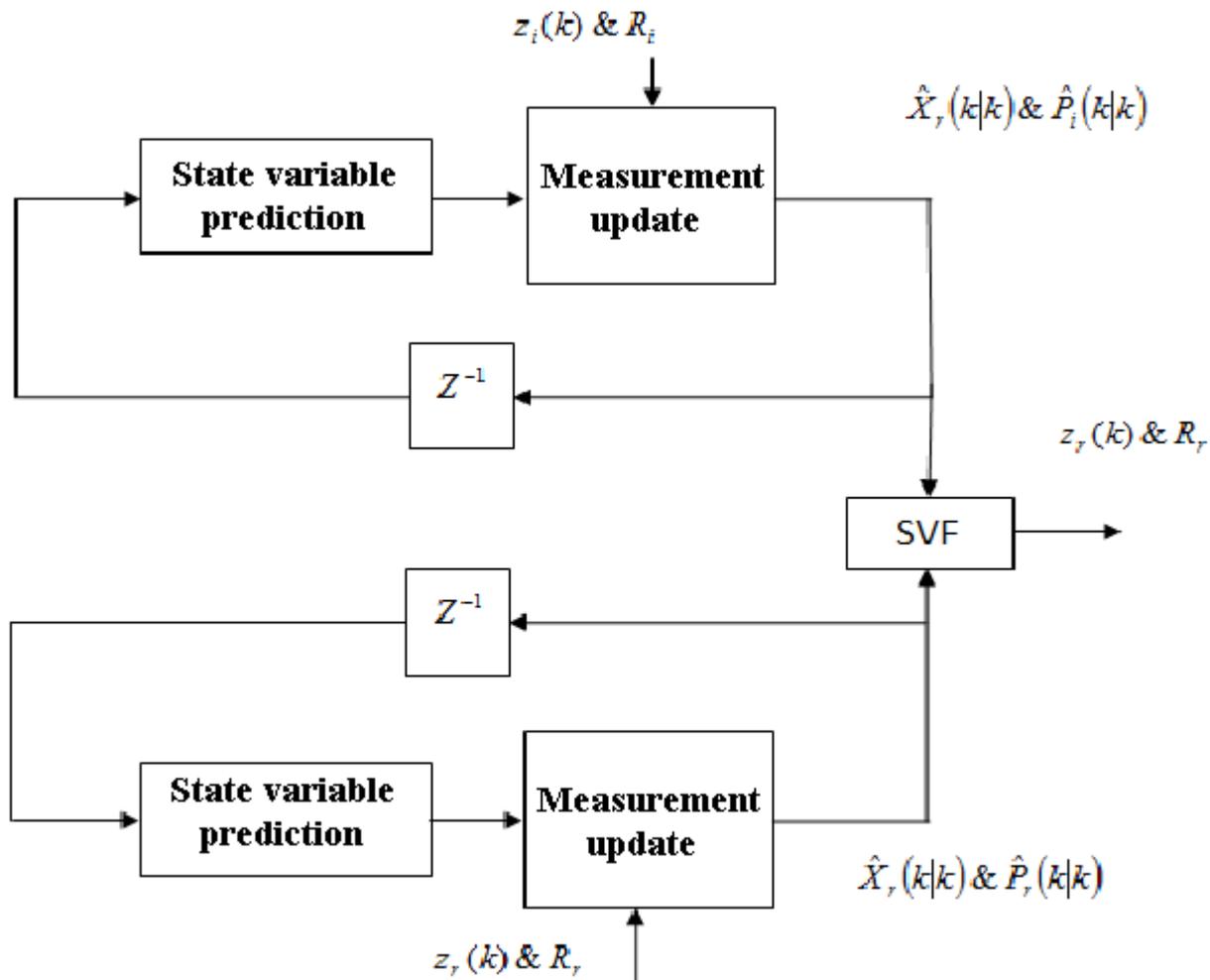


Figure 3: Block diagram of SVF algorithm

The state prediction equations in this structure are compiled as follows.

$$\begin{aligned}
 \tilde{X}_i(k|k-1) &= F\hat{X}_i(k-1|k-1) & (8) \\
 \tilde{P}_i(k|k-1) &= F\hat{P}_i(k-1|k-1)F^T + GQG^T \\
 \tilde{X}_r(k|k-1) &= F\hat{X}_r(k-1|k-1) \\
 \tilde{P}_r(k|k-1) &= F\hat{P}_r(k-1|k-1)F^T + GQG^T
 \end{aligned}$$

The measurement update equations in this structure are compiled as follows.

$$\begin{aligned}
 H_i &= h(\tilde{X}_i(k|k-1)) & (9) \\
 \tilde{z}_i(k|k-1) &= H_i\tilde{X}_i(k|k-1) \\
 e_i &= z_i(k) - \tilde{z}_i(k|k-1) \\
 S_i &= H_i\tilde{P}_i(k|k-1)H_i^T + R_i \\
 K_i &= \tilde{P}_i(k|k-1)H_i^T + S_i^{-1} \\
 \hat{X}_i(k|k) &= \tilde{X}_i(k|k-1) + K_i e_i \\
 \hat{P}_i(k|k) &= [I - K_i H_i]\tilde{P}_i(k|k-1)
 \end{aligned}$$

$$\begin{aligned}
 H_r &= h(\tilde{X}_r(k|k-1)) & (10) \\
 \tilde{z}_r(k|k-1) &= H_r \tilde{X}_r(k|k-1) \\
 e_r &= z_i(k) - \tilde{z}_r(k|k-1) \\
 S_r &= H_r \tilde{P}_r(k|k-1) H_r^T + R_r \\
 K_r &= \tilde{P}_r(k|k-1) H_r^T + S_r^{-1} \\
 \hat{X}_r(k|k) &= \tilde{X}_r(k|k-1) + K_r e_r \\
 \hat{P}_r(k|k) &= [I - K_r H_r] \tilde{P}_r(k|k-1)
 \end{aligned}$$

The composition equations in this case are formulated as follows.

$$\begin{aligned}
 \hat{X}_f(k|k) &= \hat{X}_i(k|k) + \hat{P}_i(k|k) [\hat{P}_i(k|k) + \hat{P}_r(k|k)]^{-1} (\hat{X}_r(k|k) - \hat{X}_i(k|k)) & (11) \\
 \hat{P}_f(k|k) &= \hat{P}_i(k|k) + \hat{P}_i(k|k) [\hat{P}_i(k|k) + \hat{P}_r(k|k)]^{-1} \hat{P}_i(k|k)
 \end{aligned}$$

In the feedback SVF (FSVF) algorithm shown in Figure (4), the combined state vector and state error covariance matrix are fed back to a single state predictor, and its output is used for updating and measuring.

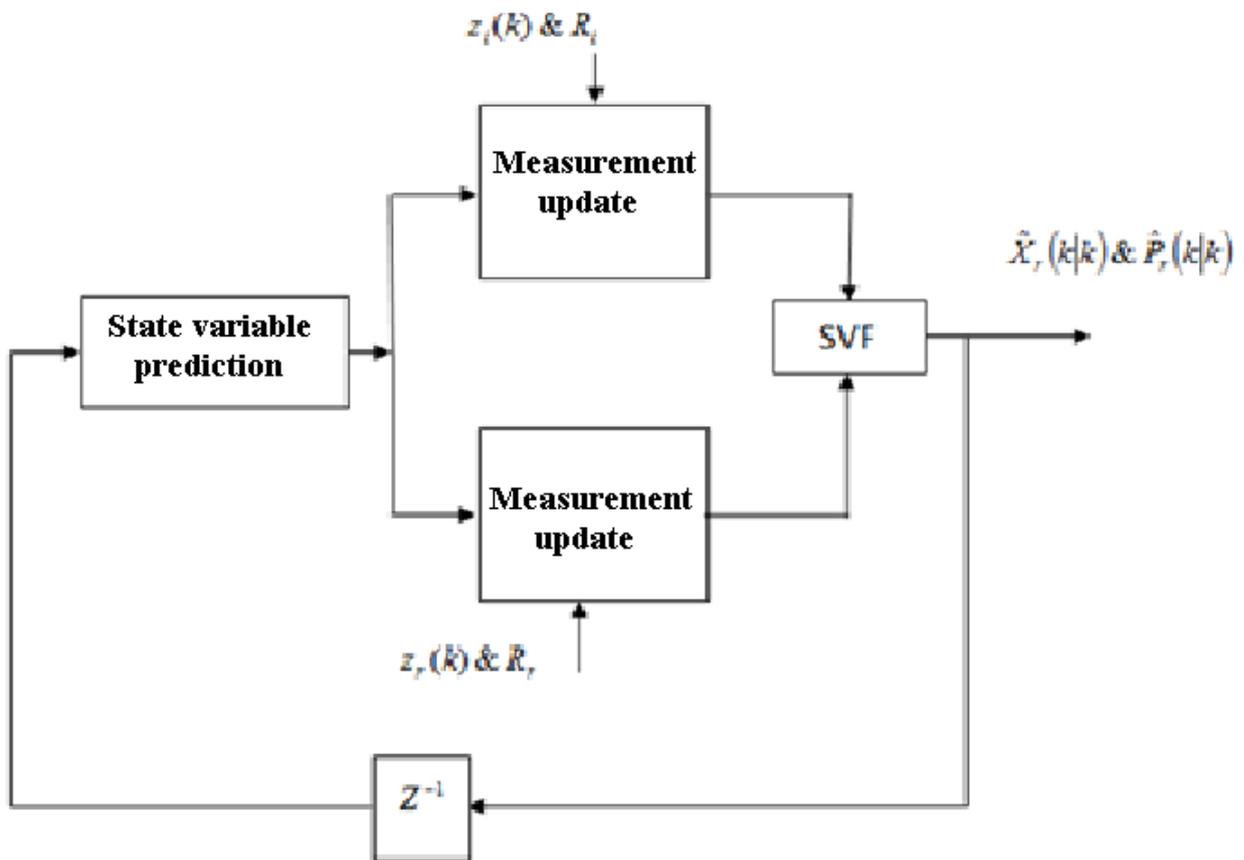


Figure 4: FSVF algorithm block diagram

As shown in Figure (5), the predicted IRST and radar state vectors are combined. Similarly, the predicted mode error covariances are combined. Combined calculations were used to

update the two measurements. IRST measurements are used in updating one of the measurements to calculate the target states. Also, radar measurements are used in updating another measurement to calculate target states.

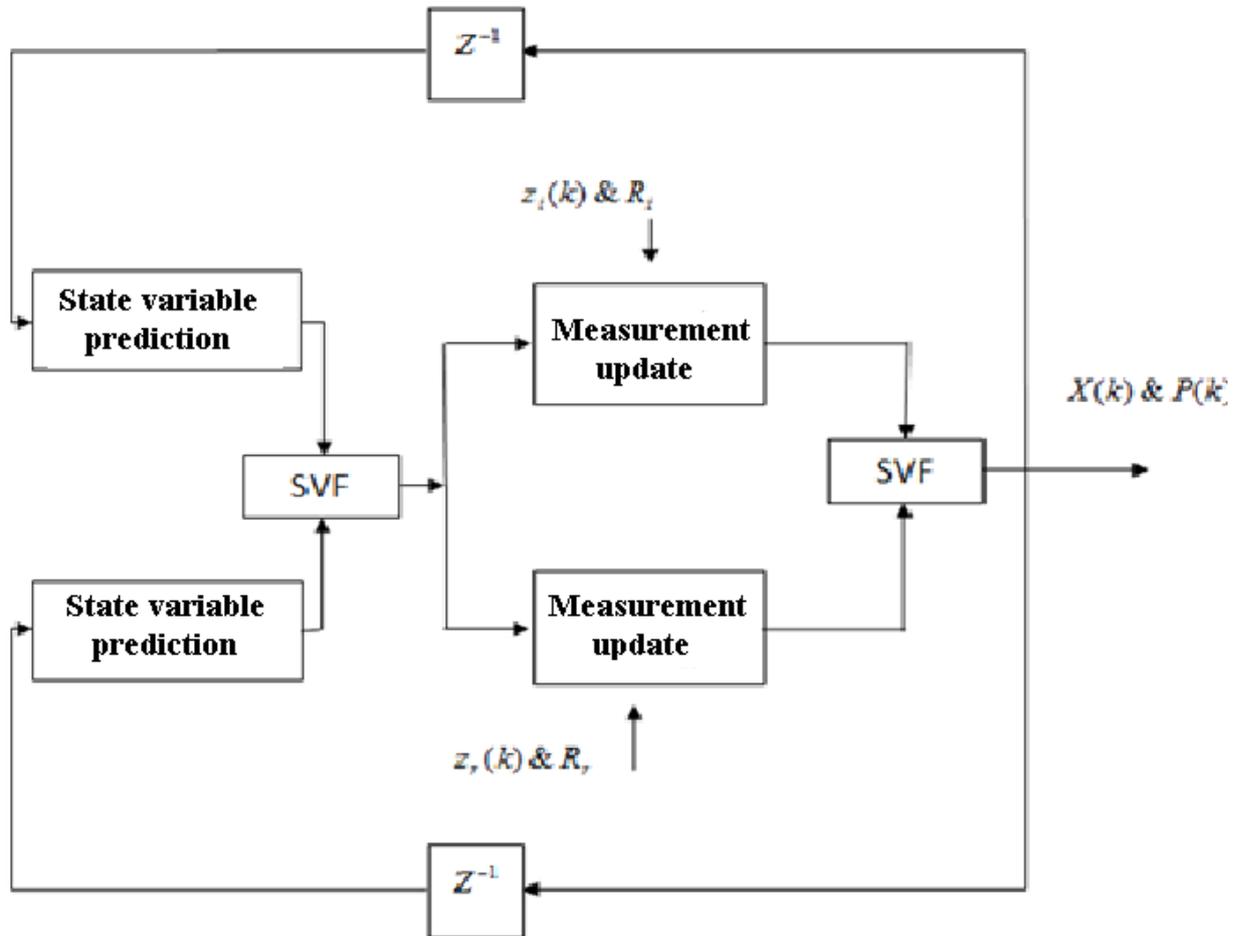


Figure 5: Block diagram of PSVF algorithm

In the decentralized Kalman filter (DKF) algorithm, the modes obtained from the local Kalman filters (LKF) were used for the global Kalman filter (GKF) for the final target calculations, as shown in Figure (6).

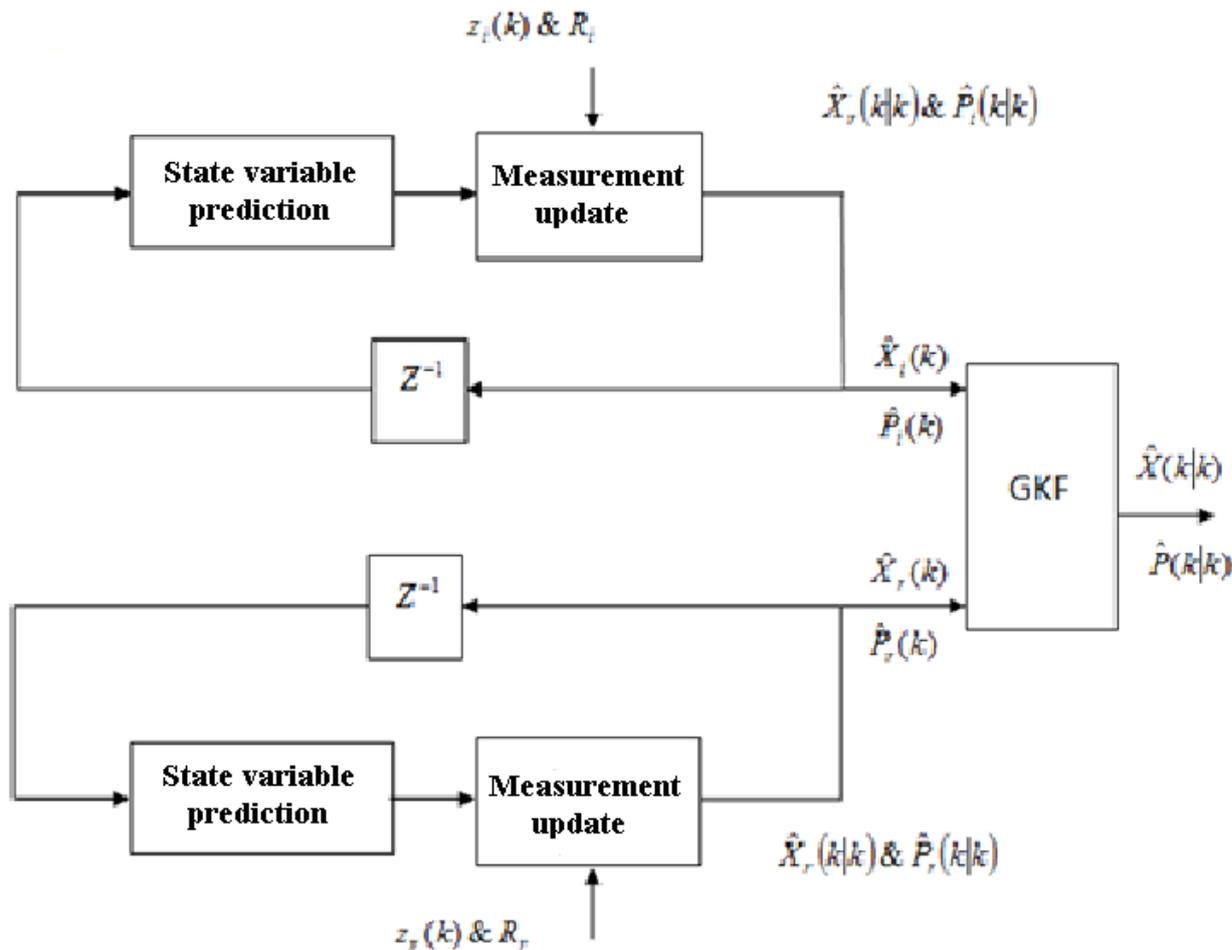


Figure 6: Block diagram of DKF algorithm

**Results**

The three-dimensional kinematic model equipped with location, speed, and acceleration components in each of the three Cartesian coordinates x, y, and z has noise gain and transfer matrices as follows.

$$\begin{aligned}
 F &= \text{diag}[\Phi \ \Phi \ \Phi] \\
 G &= \text{diag}[\zeta \ \zeta \ \zeta]
 \end{aligned}
 \tag{12}$$

where

$$\begin{aligned}
 \Phi &= \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \\
 \zeta &= [T^3/6 \quad T^2/2 \quad T]^T
 \end{aligned}
 \tag{13}$$

where T is the sampling period, F is the state transfer matrix, and G is the noise gain matrix.

The following parameters are used for simulation.

Sampling interval: 0.1 seconds

Processing noise variance: 1

The variance of measurement noise is represented in Table (1).

Table 1: Variance of measurement noise for simulation data

Sensor	Horizontal angle	Vertical angle	Range
IRST	$\sigma_{i\theta}^2 = 10^5$	$\sigma_{i\varphi}^2 = 10^5$	-----
Radar	$\sigma_{r\theta}^2 = 10^2$	$\sigma_{r\varphi}^2 = 10^2$	$\sigma_{rr}^2 = 100$

Duration of simulation: 50 seconds

Initial values:  $[x \ \dot{x} \ \ddot{x} \ y \ \dot{y} \ \ddot{y} \ z \ \dot{z} \ \ddot{z}] = [10^4 \ -200 \ 0.5 \ -1000 \ -100 \ -0.3 \ 1000 \ 1 \ 0.01]$

Simulated noise measurements for IRST data and radar data are shown in Figure (7) and Figure (8), respectively.

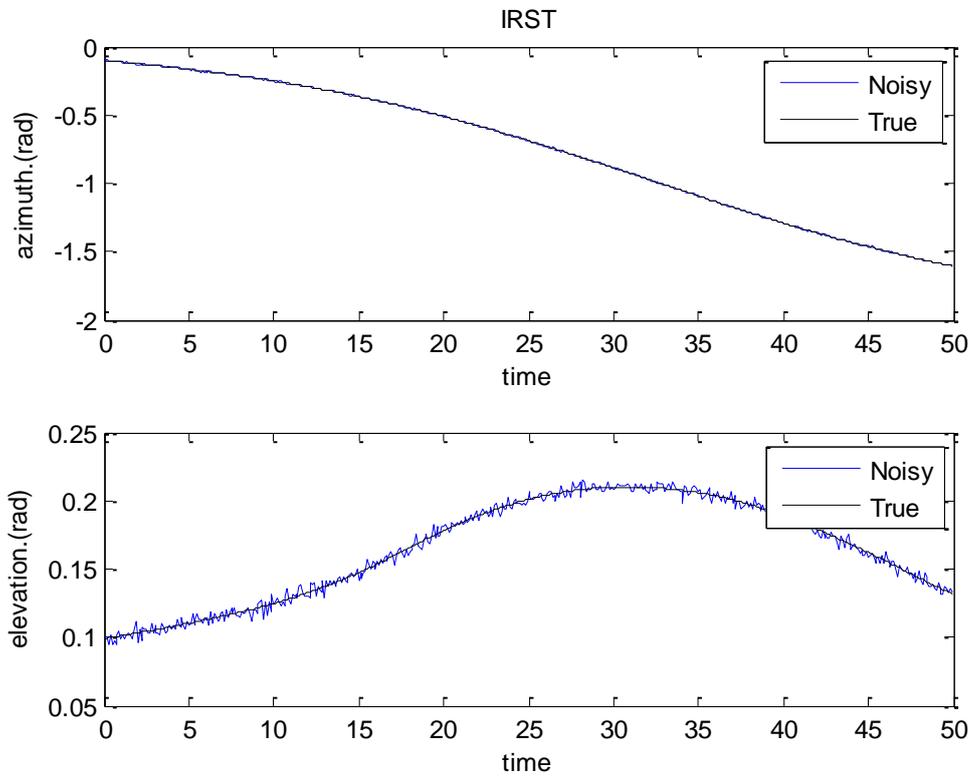


Figure 7: Simulation of noisy data for IRST

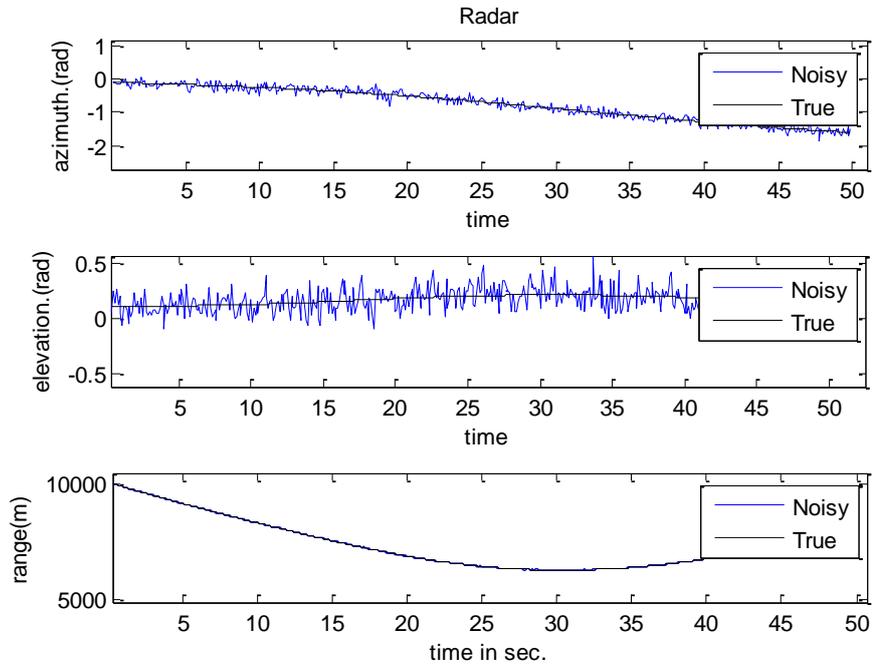


Figure 8: Simulation of noise data for radar

The initial values of the state vector are chosen as follows.

$$\hat{X}_0 = 0.9X_t \quad (14)$$

where  $\hat{X}_0$  is the estimation of the state vector and  $X$  is the real state vector is in the first scan.

The relation related to the covariance matrix of the initial state error is formulated as follows.

$$\hat{P}_0 = \text{diag}[(X_t - \hat{X}_0)^2] \quad (15)$$

The performance of the methods is compared through the following definitions.

$$PFE(x) = 100 * \frac{\text{norm}(x - \hat{x})}{\text{norm}(x)} \quad (16)$$

$$RMSPE = \sqrt{\frac{1}{N} \sum_{i=1}^N \frac{(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 + (z_i - \hat{z}_i)^2}{3}} \quad (17)$$

$$RSSPE = (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 + (z_i - \hat{z}_i)^2 \quad (18)$$

Definition of absolute error (AE)

$$AE_x(i) = |x(i) - \hat{x}(i)|, \quad i = 1, 2, \dots, N \quad (19)$$

Definition of mean absolute error (MAE)

$$MAE_x = \frac{1}{N} \sum_{i=1}^N |x(i) - \hat{x}(i)|, \quad (20)$$

The performance of MF and SVF algorithms are compared according to the above definitions in Table (2).

Table 2: Comparison of errors in the two introduced methods

Measurement	MF	SVF
PFE <sub>x</sub>	1.4739	1.4741
PFE <sub>y</sub>	1.3474	1.3501
PFE <sub>z</sub>	1.6463	1.6499
PFE <sub>xd</sub>	4.3669	4.4456
PFE <sub>yd</sub>	6.5723	6.5535
PFE <sub>zd</sub>	29.1024	28.0089
PFE <sub>xdd</sub>	130.1087	113.1093
PFE <sub>ydd</sub>	83.3922	82.3386
PFE <sub>zdd</sub>	104.0698	101.7903
MAE <sub>x</sub>	6.2477	6.4446
MAE <sub>y</sub>	9.2158	9.2963
MAE <sub>z</sub>	8.0781	8.2932
MAE <sub>xd</sub>	5.5865	5.7937
MAE <sub>yd</sub>	5.5697	5.6852
MAE <sub>zd</sub>	2.4074	2.5216
MAE <sub>xdd</sub>	0.5567	0.4921
MAE <sub>ydd</sub>	0.6958	0.6821
MAE <sub>zdd</sub>	1.1816	1.1447
RMSPE	26.8016	26.8344
RMSVE	4.2212	4.1568
RMSAE	1.0393	1.0008

RSME error for location estimation for 3 different modes is shown in Figure (9). In one case, tracking was done only based on radar data, and the resulting error was obtained. In the other two cases, the error resulting from the combination of MF and SVF algorithms has been calculated. According to this figure, using the introduced combination algorithm reduces the RSME error.

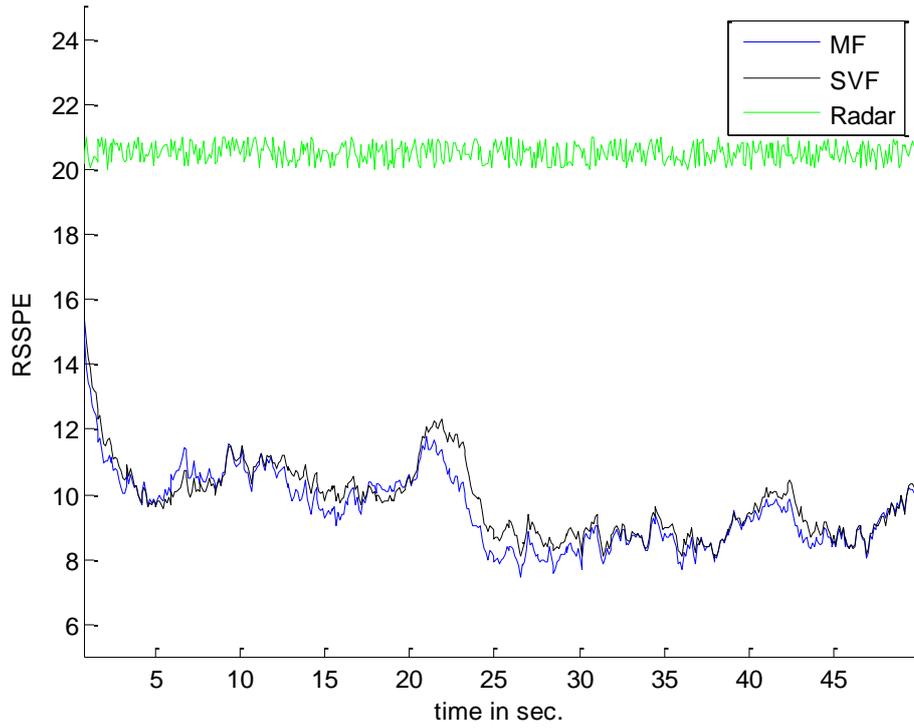


Figure 9: RSME error for location estimation

In Figure (10), the RSSE error of speed estimation for the previous three cases is represented.

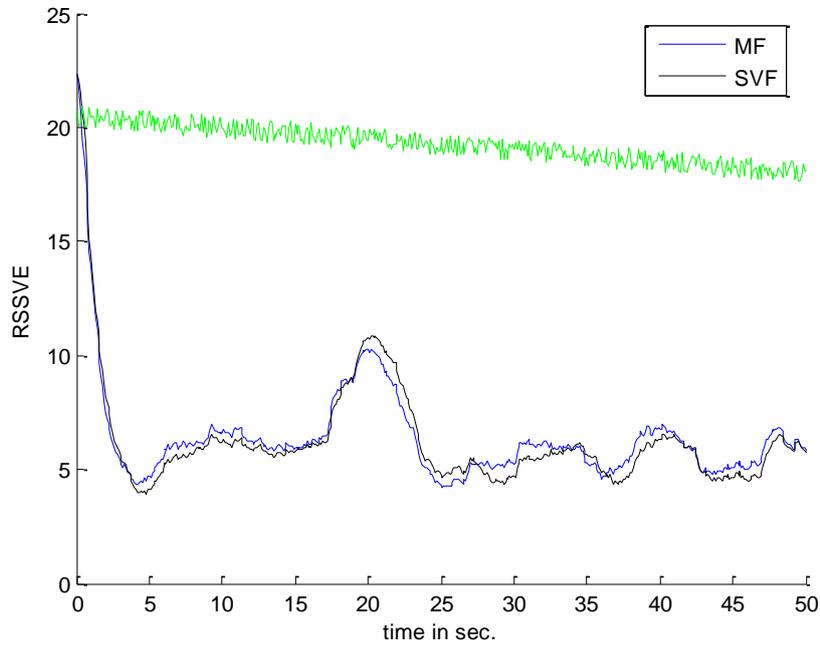


Figure 10: RSSE error for speed

## **CONCLUSION**

Since the tracking of moving targets is a vital issue in military and civilian applications, aerospace industries are always looking for accurate, low-error, computationally light, and uncomplicated algorithms to track targets. As a result, several different sensors are used in new systems for tracking. Radar systems are usually used to measure the angle and range of targets. Although they measure the range with high accuracy, radar systems cannot measure the target angle with proper accuracy. On the other hand, IRST data can measure the target angle with high accuracy and determine the direction of the target completely, but they do not provide special information about the target range.

The results of target tracking with radar or IRST are very weak compared to the combination of radar and IRST. The performance of the SVF algorithm is favorable in terms of calculation speed and implementation complexity, as expected. Also, IRST-based tracking alone is expected to require less time and subsequently show poorer performance.

According to the results, it is recommended to use a partial filter in the target tracking algorithm to implement all possible composite structures and compare them with each other.

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